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c from (1) in (3) gives

$$2b^2x(1+\cos A)+2b[my-mx-D(1+\cos A)]+2Dm-m^2y=0.$$

The condition for equal roots of b is

$$2x(1+\cos A)(2Dm-m^2y)=[my-mx-D(1+\cos A)]^2$$

$$\text{or } [D(1+\cos A)-m(x+y)]^2=2m^2xy(1-\cos A)=4m^2xy\sin^2\tfrac{1}{2}A.$$

$$\therefore\ D(1+\cos A)-m(x+y)\pm 2m\sqrt{(xy)}\sin\tfrac{1}{2}A=0.$$

$$\therefore\ x^2+2xy\cos A+y^2-\frac{m}{2\cos^2\frac{1}{2}A}[x+y\pm 2(xy)\sin\tfrac{1}{2}A]=0.$$

$$\therefore\ x^2+2xy+y^2-4xy\sin^2\tfrac{1}{2}A-\frac{m}{2\cos^2\frac{1}{2}A}[x+y\pm 2\sqrt{(xy)}\sin\tfrac{1}{2}A]=0.$$

$$\therefore\ [x\pm 2\sqrt{(xy)}\sin\tfrac{1}{2}A+y][x\mp 2\sqrt{(xy)}\sin\tfrac{1}{2}A+y-\frac{m}{2\cos^2\frac{1}{2}A}]=0.$$

$$\therefore\ x\mp 2\sqrt{(xy)}\sin\tfrac{1}{2}A+y-\frac{m}{2\cos^2\frac{1}{2}A}=0,$$

$$\text{or } x^2+2xy\cos A+y^2+[m-4m(x+y)\cos^2\tfrac{1}{2}A]/4\cos^4\tfrac{1}{2}A=0.$$

This is the circle.

CALCULUS.

144. Proposed by **G. B. M. ZERR. A. M., Ph. D.**, Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the volume of the sphere, $x^2+y^2+z^2=2az$, (a) within the paraboloid $z=Ax^2+By^2$; (b) within the cone $z^2=Ax^2+By^2$.

Solution by the PROPOSER.

$x^2+y^2+z^2=2az$....(1), $z=Ax^2+By^2$....(2), $z^2=Ax^2+By^2$....(3).

From (1), $z=a\pm\sqrt{\{a^2-x^2-y^2\}}=a\pm\sqrt{\{a^2-r^2\}}$.

From $v=\int\int zrdrd\theta$ we get

$$v=4\int_0^{\frac{1}{2}\pi}\int_0^R\sqrt{(a^2-r^2)}d\theta rdr=\tfrac{4}{3}\int_0^{\frac{1}{2}\pi}]a^3-(a^2-r^2)^{\frac{3}{2}}]d\theta.$$

(a). From (1) and (2),

$$x^2+y^2+(Ax^2+By^2)^2=2a(Ax^2+By^2).$$

$$r^2+r^4(A\cos^2\theta+B\sin^2\theta)^2=2ar^2(A\cos^2\theta+B\sin^2\theta).$$

$$\therefore r^2=\frac{2a(A\cos^2\theta+B\sin^2\theta)-1}{(A\cos^2\theta+B\sin^2\theta)^2}=R^2.$$

$$\therefore v=\frac{4a^3}{3}\int_0^{\frac{1}{2}\pi}\left[1-\left(\frac{A\cos^2\theta+B\sin^2\theta-1}{A\cos^2\theta+B\sin^2\theta}\right)^3\right]d\theta$$

$$=\frac{4a^3}{3}\int_0^{\frac{1}{2}\pi}\left[\frac{3}{A\cos^2\theta+B\sin^2\theta}-\frac{3}{(A\cos^2\theta+B\sin^2\theta)^2}+\frac{1}{(A\cos^2\theta+B\sin^2\theta)^3}\right]d\theta$$

$$=\frac{2\pi a^3}{\sqrt{AB}}\left[1-\frac{A+B}{2AB}+\frac{A^2+B^2}{8A^2B^2}\right]+\frac{\pi a^3}{6\sqrt{\{A^3B^3\}}}.$$

(*b*). From (1) and (3), $R=\dfrac{2a\sqrt{\{A\cos^2\theta+B\sin^2\theta\}}}{1+A\cos^2\theta+B\sin^2\theta}=\dfrac{2aC}{D}$.

$$\therefore v=\frac{4a^3}{3}\int_0^{\frac{1}{2}\pi}\left[1-\left(\frac{C^2-1}{D}\right)^3\right]d\theta=\frac{8a^3}{3}\int_0^{\frac{1}{2}\pi}\left(\frac{3}{D}-\frac{6}{D^2}+\frac{4}{D^3}\right)d\theta$$

$$=\frac{2\pi a^3}{\sqrt{\{(A+1)(B+1)\}}}\left[\left(\frac{A}{A+1}\right)^2+\left(\frac{B}{B+1}\right)^2\right]+\frac{4\pi a^3}{3\sqrt{\{(A+1\}^3\{B+1\}^3}}.$$

145. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the surface bounding the volume required in problem 102.

Solution by the PROPOSER.

The surface is composed of a portion of the surface of the cone, and a portion of the surface of the paraboloid.

The equation to the cone is $x^2+y^2=c^2z^2$.

The equation to the paraboloid is $y^2+z^2=4a(a+x)$.

$S=S_c+S_p$.

$$S_c=\frac{2}{c}\int_{x_2}^{x_1}\int_0^{y_1}\sqrt{\{1+c^2\}}dxdy,\ \{\text{see 102 for limits}\}$$

$$=\frac{2}{c}\int_{x_2}^{x_1}\sqrt{\{4ac^2(a+x)-x^2\}}dx.$$

Let $x=2ac^2-2ac\sqrt{\{1+c^2\}}\cos\theta$.

$$\therefore S_c=8a^2c(1+c^2)\int_0^{\pi}\sin^2\theta d\theta=4\pi a^2c(1+c^2).$$

$$S_p=4\int_{x_2}^{x_1}\int_0^{y_1}\sqrt{\frac{a(2a+x)}{4a(a+x)-y^2}}dxdy=4\int_{x_2}^{x_1}\sqrt{\{a(2a+x)\}}\sin^{-1}\sqrt{\frac{4ac^2(a+x)-x^2}{4a(1+c^2)(a+x)}}dx$$